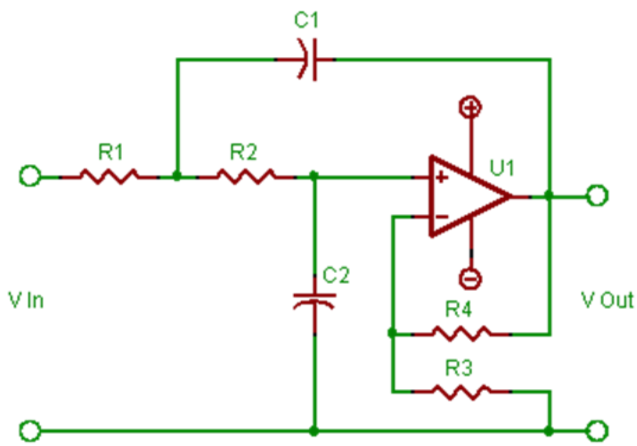
May 2012

ECE 232 – Advanced Electrical Circuit Analysis

Lab6

Preliminary Work:

1. Second order low-pass filter



$c_1 = c_2 = 0.1\mu F$

$R_1 = R_2 = 1.2k\Omega$

$R_3 = R_4 = 1k\Omega$

$V_- = V_o \frac{R_3}{R_3+R_4} = \frac{V_o}{2} = V_+$ in linear mode

1^{st} node equation for $V_+ = \frac{V_o}{2}$;

$$\frac{V_o/2}{1/sC_2} + \frac{V_o/2 - V_x}{R_2} = 0$$

$$V_x = V_o\left(\frac{sR_2C_2}{2} + \frac{1}{2}\right) \qquad (eqn - 1)$$

2^{nd} node equation for V_x ;

$$\frac{V_x - V_o/2}{R_2} + \frac{V_x - V_o}{1/sC_1} + \frac{V_x - V_i}{R_1} = 0$$

$$\frac{V_i}{R_1} = V_x\left(\frac{1}{R_1} + sC_1 + \frac{1}{R_2}\right) - V_o\left(sC_1 + \frac{1}{2R_2}\right)$$

From eqn-1

$$\frac{V_i}{R_1} = V_o\left(\frac{sR_2C_2}{2} + \frac{1}{2}\right)\left(\frac{1}{R_1} + sC_1 + \frac{1}{R_2}\right) - V_o\left(sC_1 + \frac{1}{2R_2}\right)$$

$$\frac{V_i}{R_1} = V_o\left(\frac{sR_2C_2}{2R_1} + \frac{1}{2R_1} + \frac{s^2R_2C_2C_1}{2} + \frac{sC_1}{2} + \frac{sC_2}{2} + \frac{1}{2R_2} - sC_1 - \frac{1}{2R_2}\right)$$

$$V_i = V_o\left(\frac{sR_2C_2}{2} + \frac{1}{2} + \frac{s^2R_2C_2C_1}{2} - \frac{sC_1}{2} + \frac{sC_2}{2}\right)$$

$$\frac{V_o(s)}{V_i(s)} = \frac{2}{sR_2C_2 + 1 + s^2R_1R_2C_1C_2 - sR_1C_1 + sR_1C_2}$$

$$c_1 = c_2$$

$$R_1 = R_2$$

$$\frac{V_o(jw)}{V_i(jw)} = \frac{2}{1 - w^2R_1R_2C_1C_2 + jwR_1C_2} = \frac{2}{1 - w^2 * 14.4 * 10^{-9} + jw0.12 * 10^{-3}}$$

$$|H(jw)| = \frac{2}{\sqrt{(1 - w^2 * 14.4 * 10^{-9})^2 + (w * 0.12 * 10^{-3})^2}}$$

$$|H(jw)| = \frac{|H(0)|}{\sqrt{2}} = \frac{2}{\sqrt{2}}$$

$$(1 - w^2 * 14.4 * 10^{-9})^2 + (w * 0.12 * 10^{-3})^2 = 0$$

$$w_c \cong 10600 rad/sec$$

$$f_c \cong 1687 Hz$$

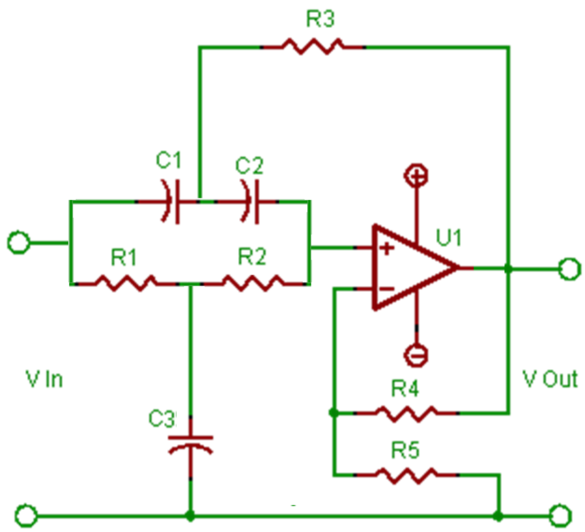
Note! If we define w_c as; $|H(jw_c)| = \frac{|H(jw_c)|_{max}}{\sqrt{2}} = \frac{2.3}{\sqrt{2}}$

$$w_c \cong 9800 rad/sec$$

$$f_c \cong 1560 Hz$$

$$\angle H(jw) = -\tan^{-1}\left[\frac{w * 0.12 * 10^{-3}}{1 - w^2 * 14.4 * 10^{-9}}\right]$$

2. Band-stop filter



$$c_1 = c_2 = 0.5\mu F$$

$$c_3 = 1\mu F$$

$$R_1 = R_2 = 560\Omega$$

$$R_3 = 270\Omega$$

$$R_4 = 1k\Omega$$

$$R_5 = 2.2k\Omega$$

$$H(s) = K\frac{s^2 + \frac{1}{R_1R_2C_1C_2}}{s^2 + s\left[\frac{1-K}{R_3C_1} + \frac{1}{R_2}\left(\frac{1}{C_1} + \frac{1}{C_2}\right)\right] + \frac{1}{R_1R_2C_1C_2}}$$

$$K = 1 + \frac{R_4}{R_5} \cong 1.45$$

$$H(s) = 1.45\frac{s^2 + 12.76 * 10^6}{s^2 + s * 3776 + 12.76 * 10^6}$$

$$H(jw) = 1.45\frac{12.76 * 10^6 - w^2}{(12.76 * 10^6 - w^2) + jw * 3776}$$

$$|H(jw)| = 1.45\frac{12.76 * 10^6 - w^2}{(12.76 * 10^6 - w^2)^2 + (w * 3776)^2}$$

$$\angle H(jw) = -\tan^{-1}\left[\frac{3776w}{12.76 * 10^6 - w^2}\right]$$

$$|H(0)| = 1.45 \quad ==> \quad |H(jw)| = \frac{1.45}{\sqrt{2}} \cong 1$$

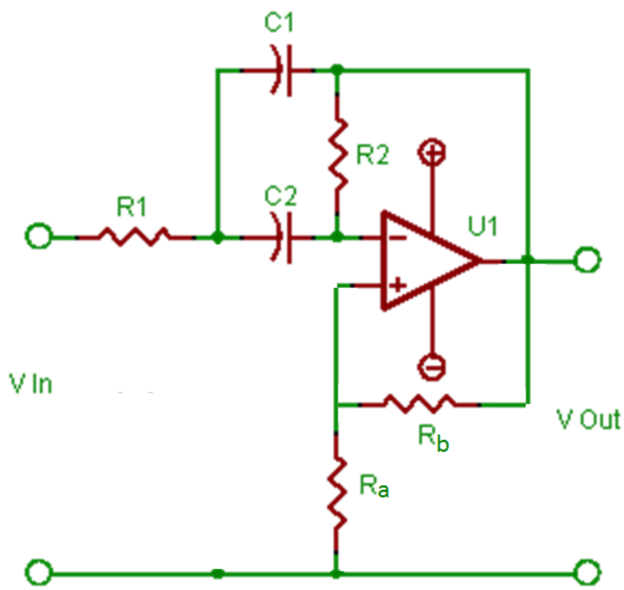
$$|H(jw)| = 1 ==> \quad w_{c_1} \cong 2185 rad/sec$$

$$w_{c_2} \cong 5840 rad/sec$$

$$|H(jw_o)| = 0 ==> 12.76 * 10^6 - {w_o}^2 = 0 ==> w_o \cong 3571 rad/sec$$

3. Band-pass filter

a.



$c_1 = c_2 = 4.7\text{nF}$

$R_1 = 1\text{k}\Omega$

$R_2 = 2.2\text{k}\Omega$

$R_a = R_b = 10\text{k}\Omega$

$V_+ = V_o \frac{R_a}{R_a + R_b} = \frac{V_o}{2} = V_-$ in linear mode

1^{st} node equation for $V_- = \frac{V_o}{2}$;

$$\frac{{V_o}/{2} - V_o}{R_2} + \frac{{V_o}/{2} - V_x}{{1}/{sC_2}} = 0$$

$$V_x = V_o\left(\frac{1}{2} - \frac{1}{2sR_2C_2}\right) \qquad (eqn - 1)$$

2^{nd} node equation for V_x ;

$$\frac{V_x - {V_o}/{2}}{{1}/{sC_1}} + \frac{V_x - V_o}{{1}/{sC_1}} + \frac{V_x - V_i}{R_1} = 0$$

$$\frac{V_i}{R_1} = V_x\left(\frac{1}{R_1} + sC_1 + sC_2\right) - V_o\left(sC_1 + \frac{sC_2}{2}\right)$$

$$\frac{V_i}{R_1} = V_o\left(\frac{1}{2} - \frac{1}{2sR_2C_2}\right)\left(\frac{1}{R_1} + sC_1 + sC_2\right) - V_o\left(sC_1 + \frac{sC_2}{2}\right)$$

$$\frac{V_i}{R_1} = V_o\left(\frac{1}{2R_1} - \frac{1}{2sR_1R_2C_2} + \frac{sC_1}{2} - \frac{C_1}{2R_2C_2} + \frac{sC_2}{2} - \frac{1}{2R_2} - sC_1 - \frac{sC_2}{2}\right)$$

$$c_1 = c_2$$

$$V_i = V_o\left(\frac{sR_2C_2 - sR_1C_1 - sR_1C_2 - 1 - s^2R_2{C_2}^2R_1}{2sR_1R_2C_2}\right)$$

$$\frac{V_o(s)}{V_i(s)} = \frac{-2sR_2C_2w}{\left(s^2{C_2}^2R_1R_2\right)^2 + (sR_1C_1 + sR_1C_2 - sR_2C_2 + 1)^2}$$

$$\frac{V_o(jw)}{V_i(jw)} = \frac{-j2R_2C_2w}{\left(1 - w^2{C_2}^2R_1R_2\right)^2 + (jw2R_1C_1 - jwR_2C_2)^2}$$

$$|H(jw)| = \frac{2wR_2C_2}{\sqrt{\left(1 - w^2{C_2}^2R_1R_2\right)^2 + w^2(2R_1C_1 - R_2C_2)^2}}$$

$$\angle H(jw) = -90 - \tan^{-1}\left[\frac{-w * 0.94 * 10^{-6}}{1 - w^2 * 4.86 * 10^{-11}}\right]$$

The standard form of the transfer function of a second order band-pass filter is as follows

$$\frac{Ks}{\left(\frac{s}{w_o}\right)^2 + 2\alpha\frac{s}{w_o} + 1}$$

$$w_o = \frac{1}{C_1\sqrt{R_1R_2}} ==> \ w_o \cong 143.4\ krad/sec$$

$$at \quad w_o, |H(jw_o)| \cong 22$$

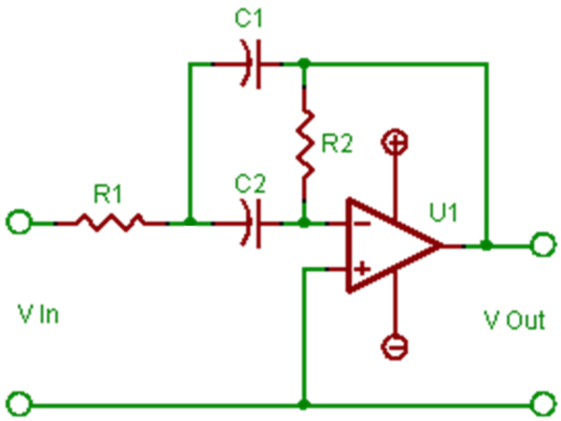
$$|H(jw_c)| = \frac{22}{\sqrt{2}} = 15.5$$

$$w_{c_1} \cong 134 krad/sec$$

$$w_{c_2} \cong 154 krad/sec$$

$$\Delta_w = w_{c_2} - w_{c_1} = 20 krad/sec$$

b. in linear region $V_+ = V_- = 0$



1^{st} node equation for V_- ;

$$\frac{V_- - V_o}{R_2} + \frac{V_- - V_x}{1/sC_2} = 0$$

$$V_x = -\frac{V_o}{sR_2C_2} \qquad (eqn - 1)$$

2^{nd} node equation for V_x ;

$$\frac{V_x - V_+}{1/sC_1} + \frac{V_x - V_o}{1/sC_1} + \frac{V_x - V_i}{R_1} = 0$$

$$\frac{V_i}{R_1} = V_x\left(\frac{1}{R_1} + sC_1 + sC_2\right) - V_o(sC_1)$$

$$\frac{V_i}{R_1} = -V_o\left(\frac{1}{R_2} + \frac{1}{sR_1R_2C_2} + \frac{C_1}{R_2C_2} + sC_1\right)$$

$$V_i = -V_o\left(\frac{sR_1C_1 + 1 + sR_1C_2 + s^2R_2R_1C_2C_1}{sR_1R_2C_2}\right)$$

$$\frac{V_o(s)}{V_i(s)} = \frac{-sR_2C_2}{s^2C_1C_2R_1R_2 + sR_1C_1 + sR_1C_2 + 1}$$

$$H(jw) = \frac{-jwR_2C_2}{(1 - w^2C_1C_2R_1R_2) + jw(R_1C_1 - R_2C_2)}$$

$$H(jw) = \frac{-jw * 10.34 * 10^{-6}}{(1 - w^2 * 4.86 * 10^{-11}) + jw * 9.4 * 10^{-6}}$$

$$|H(jw)| = \frac{-jw * 10.34 * 10^{-6}}{\sqrt{(1 - w^2 * 4.86 * 10^{-11})^2 + (w * 9.4 * 10^{-6})^2}}$$

$$\angle H(jw) = -90 - \tan^{-1}\left[\frac{w * 9.4 * 10^{-6}}{1 - w^2 * 4.86 * 10^{-11}}\right]$$

The standard form of the transfer function of a second order filter:

$$\frac{Ks}{\left(\frac{s}{w_o}\right)^2 + 2\alpha\frac{s}{w_o} + 1}$$

$$w_o \cong 143.4\ krad/sec$$

$$at \quad w_o, |H(jw_o)| \cong 1.1$$

$$|H(jw_c)| = \frac{1.1}{\sqrt{2}}$$

$$w_{c_1} \cong 76krad/sec$$

$$w_{c_2} \cong 270krad/sec$$

$$\Delta_w = w_{c_2} - w_{c_1} = 194krad/sec$$