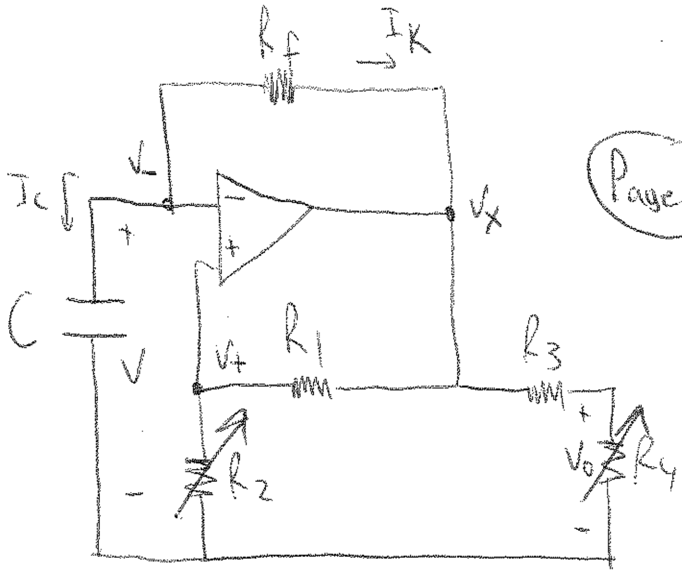


Page 1

$V = V_-$

$V_+ = V_x \dfrac{R_2}{R_1+R_2}$

$V_0 = V_x \dfrac{R_4}{R_3+R_4}$

$C = 1\,\mu F$
$R_F = 0.5\,k\Omega$
$R_1 = 2.2\,k\Omega$
$R_3 = 1\,k\Omega$
$R_2, R_4 \rightarrow$ variable

if $V_+ > V_-$ $\quad V_x = 12 \quad V_+ = V_x \dfrac{R_2}{R_1+R_2} = 12 \dfrac{R_2}{R_1+R_2}$

if $V_- > V_+$ $\quad V_x = -12 \quad V_+ = V_x \dfrac{R_2}{R_1+R_2} = -12 \dfrac{R_2}{R_1+R_2}$

limit values for V_+

$I_C = -I_K$

$$C\frac{dV}{dt} = -\left[\frac{V - V_x}{R_F}\right] \Rightarrow C\frac{dV}{dt} + \frac{V}{R_F} = \frac{V_x}{R_F} \qquad \boxed{\frac{dV}{dt} + \frac{V}{CR_F} = \frac{V_x}{CR_F}}$$

differential equation

* Assume, $V = V_- = -12\dfrac{R_2}{R_1+R_2}$ and $V_+ = 12\dfrac{R_2}{R_1+R_2} \Rightarrow V_+ > V_- \Rightarrow V_x = 12$
$t=0$

then $\boxed{\dfrac{dV}{dt} + \dfrac{V}{CR_F} = \dfrac{12}{CR_F} \quad V(0) = -12\dfrac{R_2}{R_1+R_2}}$ $\xrightarrow{\text{solution}}$ $\boxed{V = -12\dfrac{R_2}{R_1+R_2}e^{-\frac{t}{CR_F}} + 12\left(1 - e^{-\frac{t}{CR_F}}\right)}$ for $0 < t < t_1$

~~let~~ at $V(t_1) = 12\dfrac{R_2}{R_1+R_2} = V_+ \Rightarrow$

$$V(t_1) = 12\frac{R_2}{R_1+R_2} = -12\frac{R_2}{R_1+R_2}e^{-\frac{t_1}{CR_F}} + 12\left[1 - e^{-\frac{t_1}{CR_F}}\right]$$

$$12\,e^{-\frac{t_1}{CR_F}}\left[1 + \frac{R_2}{R_1+R_2}\right] = 12\frac{R_1}{R_1+R_2} \Rightarrow e^{-\frac{t_1}{CR_F}} = \frac{R_1}{R_1+2R_2}$$

$$-\frac{t_1}{CR_F} = \ln\frac{R_1}{R_1+2R_2} \qquad \frac{t_1}{CR_F} = \ln\frac{R_1+2R_2}{R_1}$$

$$\boxed{t_1 = CR_F \ln\frac{2R_2+R_1}{R_1}}$$

** at $t=t_1$ $V_- = 12\frac{R_1}{R_1+R_2} \geq V_+$ hence V_x becomes -12 Volt $\left(V_x=-12 \text{ Volt}\right)$

if $V_x=-12 \Rightarrow V_+ = -12\frac{R_2}{R_1+R_2}$

then $\boxed{\frac{dV}{dt} + \frac{V}{CR_F} = \frac{-12}{CR_F} \qquad V(t_1) = 12\frac{R_2}{R_1+R_2}}$ $\xrightarrow{\text{solution}}$ $\boxed{V = 12\frac{R_2}{R_1+R_2}e^{-\frac{(t-t_1)}{R_FC}} - 12\left[1-e^{-\frac{(t-t_1)}{CR_F}}\right] \quad \text{for } t_1 \leq t < t_2}$

Let at $V(t_2) = -12\frac{R_2}{R_1+R_2} = V_+$

$$V(t_2) = -12\frac{R_2}{R_1+R_2} = 12\frac{R_2}{R_1+R_2}e^{-\frac{(t_2-t_1)}{CR_F}} - 12\left[1-e^{-\frac{(t_2-t_1)}{CR_F}}\right]$$

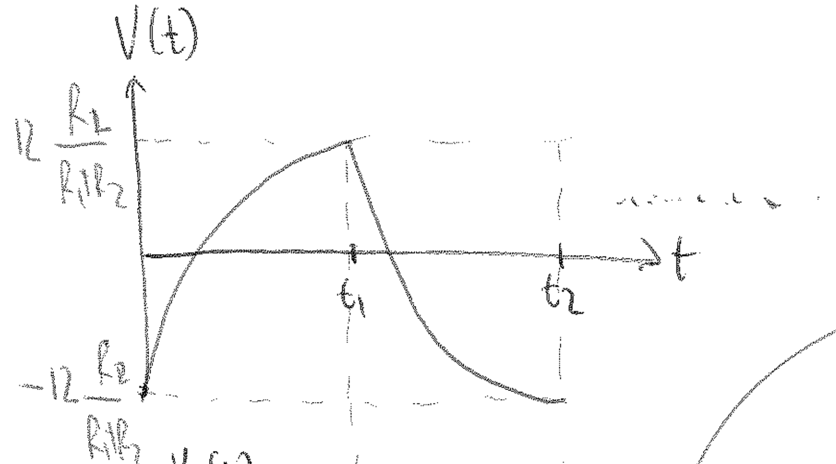
$$12\left[1-\frac{R_2}{R_1+R_2}\right] = 12e^{-\frac{(t_2-t_1)}{CR_F}}\left[\frac{R_2}{R_1+R_2}+1\right]$$

$$\frac{R_1}{R_1+R_2} = e^{-\frac{(t_2-t_1)}{CR_F}}\frac{2R_2+R_1}{R_1+R_2}$$

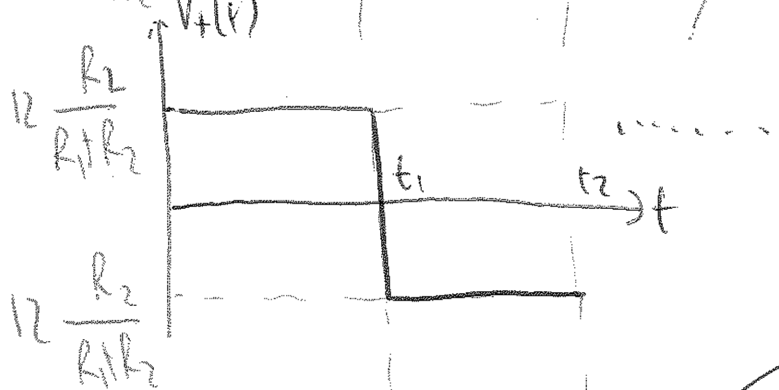
$$-\frac{(t_2-t_1)}{CR_F} = \ln\frac{R_1}{R_1+2R_2} \qquad \frac{t_2-t_1}{CR_F} = \ln\frac{R_1+2R_2}{R_1}$$

$$t_2 = t_1 + CR_F\ln\frac{R_1+2R_2}{R_1} \Rightarrow t_2 = 2t_1$$

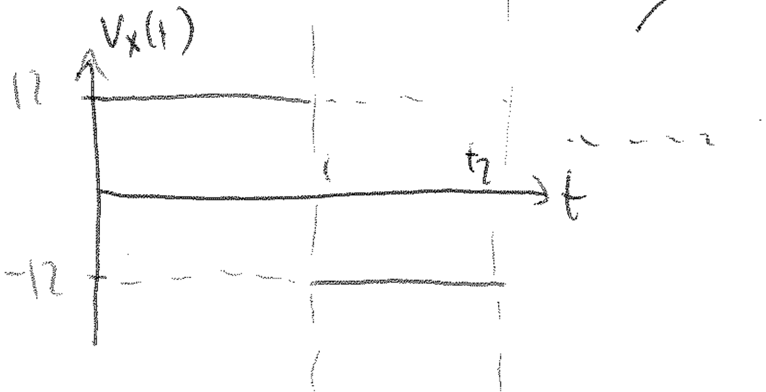
Let's draw V, V_+, V_x and V_0 when $0 < t < t_2$; then these signals will continue periodically with period t_2



As seen V_+ is a square wave (oscillator)

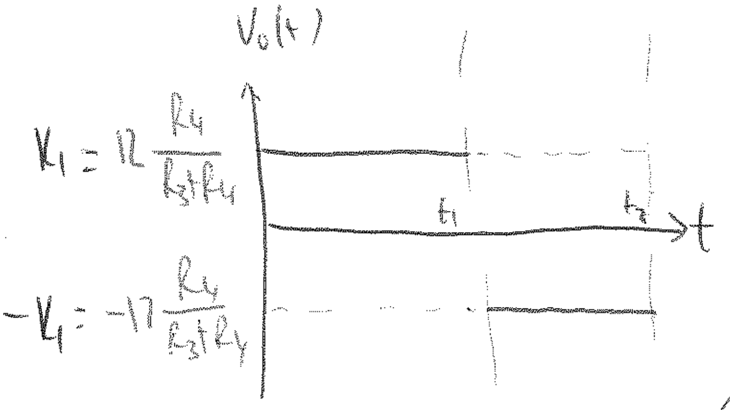


$V_x(t)$ is a square wave (oscillator)



$$V_0(t) = V_x(t)\frac{R_4}{R_3+R_4}$$

another square wave (oscillator)



$$t_2 = 2t_1 = 2CR_F \ln\frac{R_1+2R_2}{R_1} = T$$

$$\boxed{\begin{array}{l} T = \text{period of } V_0(t) = t_2 \\ K_1 = 12\frac{R_4}{R_3+R_4} = \text{amplitude of } V_0(t) \end{array}}$$

$V_0(t)$ is a square oscillator

assume $K_1 = 2$ Volt [amplitude of square wave $V_0(t)$]

$$K_1 = 12 \frac{R_4}{R_3 + R_4} = 2 \qquad \frac{R_4}{R_3 + R_4} = \frac{1}{6} \qquad 6R_4 = R_3 + R_4 \quad 5R_4 = R_3 \rightarrow R_4 = \frac{R_3}{5}$$

$$R_4 = \frac{1k\Omega}{5} = 200\Omega$$

(R_4 should be adjusted such that $R_4 \approx 200\Omega$)
(use $1k\Omega$ pot for R_4)

assume $f = 500$ Hertz $\quad T = \frac{1}{f} = \frac{1}{500} = t_2 = 2\,C R_F \ln \frac{R_1 + 2R_2}{R_1}$

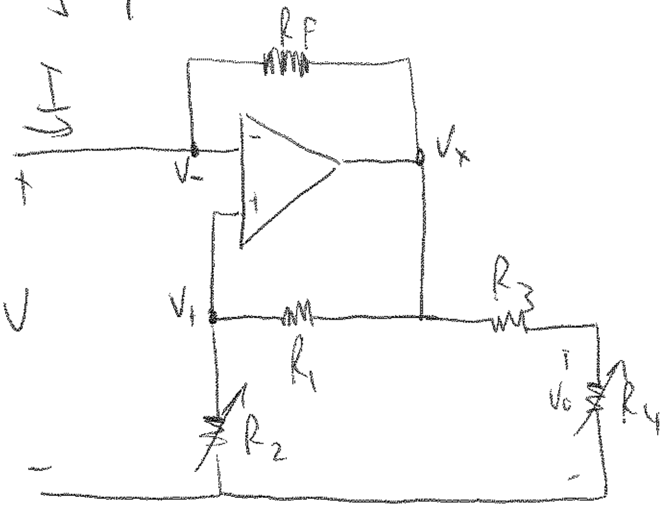
$$\frac{1}{500} = 2 \times 10^{-6} \times 0.5 \times 10^{3} \times \ln \frac{2.2 \times 2R_2}{2.2}$$

$$2 = \ln \frac{2.2 + 2R_2}{2.2} \qquad e^2 = \frac{2.2 + 2R_2}{2.2} \qquad 2.2e^2 - 2.2 = 2R_2$$

$$R_2 = \frac{2.2[e^2 - 1]}{2} = 1.1\,[e^2 - 1]\ k\Omega = 7.026\ k\Omega$$

(For R_2 use $10k\Omega$ pot)

Driving point characteristics



* if $V_x = 12$ $\quad V_+ = 12\dfrac{R_2}{R_1+R_2}$ assume $V = V_- < V_+$ but increase $[V = V_-]$

$$I = \frac{V - V_x}{R_F} = \frac{V-12}{R_F}$$

[The current value when $V = V_- = 12\dfrac{R_2}{R_1+R_2}$

$$I = \frac{12\frac{R_2}{R_1+R_2} - 12}{R_F} = -12\frac{R_1}{R_F(R_1+R_2)} \text{ Ampere}$$]

when $V > 12\dfrac{R_2}{R_1+R_2}$ $\quad V_- > V_+$ and hence $V_x = -12 \Rightarrow V_+ = -12\dfrac{R_2}{R_1+R_2}$

then $I = \dfrac{V - V_x}{R_F} = \dfrac{V-(-12)}{R_F} = \dfrac{V+12}{R_F}$

** if $V_x = -12$ $\quad V_+ = -12\dfrac{R_2}{R_1+R_2}$ assume $V = V_- > V_+$ but decrease $[V = V_-]$

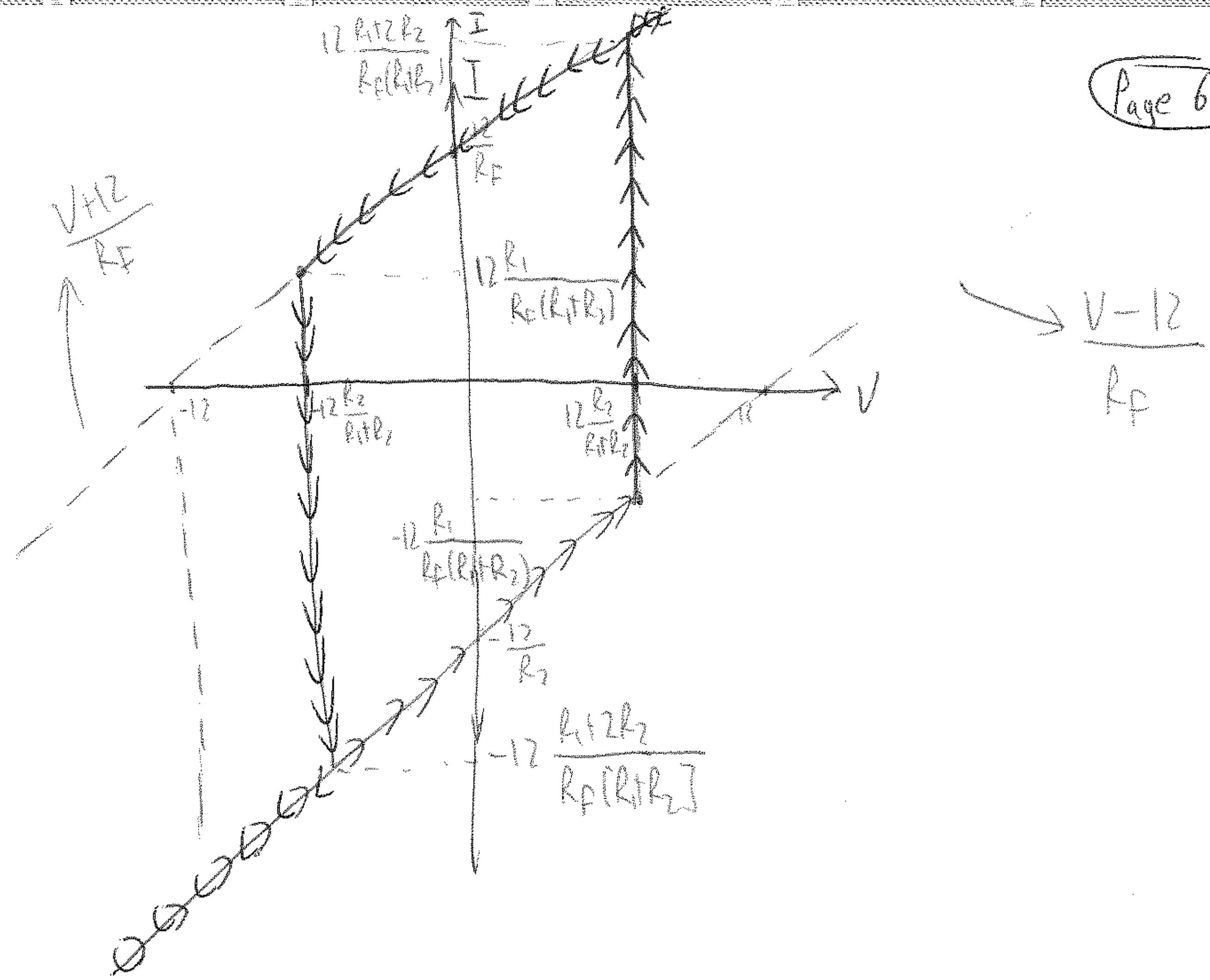
$$I = \frac{V - V_x}{R_F} = \frac{V-(-12)}{R_F} = \frac{V+12}{R_F}$$

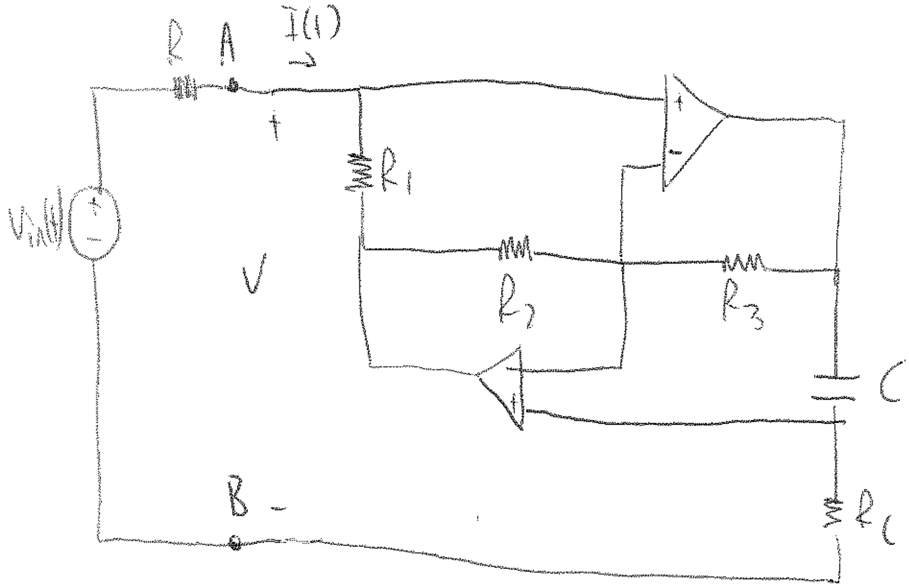
[The current value $V = V_- = -12\dfrac{R_2}{R_1+R_2}$

$$I = \frac{-12\frac{R_2}{R_1+R_2} + 12}{R_F} = \frac{12R_1}{R_F(R_1+R_2)} \text{ Ampere}$$]

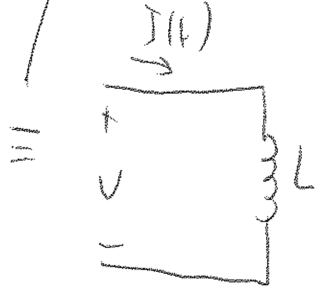
when $V < -12\dfrac{R_2}{R_1+R_2}$ $\quad V_- < V_+$ and hence $V_x = +12 \Rightarrow V_+ = 12\dfrac{R_2}{R_1+R_2}$

then $I = \dfrac{V - V_x}{R_F} = \dfrac{V-12}{R_F}$


I
$12\frac{R_1+2R_2}{R_f(R_1+R_2)}$
$\frac{12}{R_f}$
$12\frac{R_1}{R_f(R_1+R_2)}$
$\frac{V+12}{R_f}$
-12
$-12\frac{R_2}{R_1+R_2}$
$12\frac{R_2}{R_1+R_2}$
V
12
$-12\frac{R_1}{R_f(R_1+R_2)}$
$-\frac{12}{R_f}$
$-12\frac{R_1+2R_2}{R_f(R_1+R_2)}$
$\frac{V-12}{R_f}$



equivalence



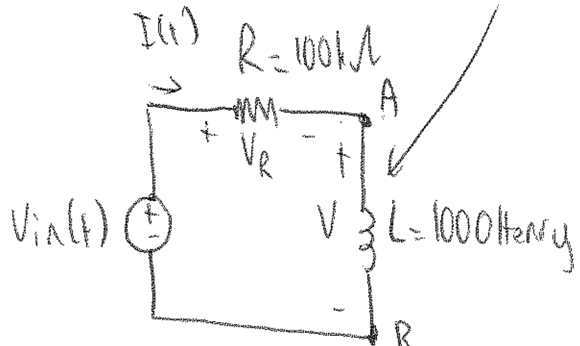
$$L = \frac{R_1 R_3 R_L C}{R_2}$$

$$V = L\frac{dI}{dt}$$

$R_1 = R_2 = R_3 = R_L = 10k\Omega$

$C = 10\mu F$

$$L = \frac{10\times10^3 \times 10\times10^3 \times 10\times10^3 \times 10\times10^{-6}}{10\times10^3} = 1000 \text{ Henry}$$



$$V_{in} = V_R + V$$

$$V_{in} = 10^5 \times I + 10^3 \frac{dI}{dt}$$

$$\boxed{\frac{dI}{dt} + 100I = \frac{V_{in}}{1000}} \Rightarrow \underbrace{\tau}_{\text{time constant}} = \frac{L}{R} = \frac{1000}{10^5} = 0.01$$

Vin(t) ⇒ Square wave
with 2 Volt peak-to-peak
value and 1 volt DC
offset with frequency = 10 Hertz

f = 10 Hertz T = 0.1 sec

$$\underbrace{T}_{\text{period of square wave}} >> \underbrace{\tau}_{\text{time constant}}$$ (hence system goes to steady-state)

$0 < t < \frac{T}{2}$, $V_{in}(t) = 2$ Volt $\left[1 \xrightarrow{} \text{peak value} + 1 \longrightarrow \text{DC offset} \right]$, $\boxed{I_0 = \frac{0}{10^5} = 0 \text{ Ampere} \quad I_f = \frac{2}{10^5} \text{ Ampere}}$

$$I(t) = -\frac{2}{10^5} e^{-\frac{t}{\tau}} + \frac{2}{10^5}\left[1 - e^{-\frac{t}{\tau}}\right]$$

$\frac{T}{2} < t < T$, $V_{in}(t) = -2$ Volt $\left[-1 \xrightarrow{} \text{negative peak} + 1 \rightarrow \text{DC offset} \right]$, $\boxed{I_0 = \frac{2}{10^5} \text{ Ampere} \quad I_f = 0 \text{ Ampere}}$

$$I(t) = \frac{2}{10^5} e^{-\frac{(t - \frac{T}{2})}{\tau}} + 0\left[1 - e^{-\frac{t}{\tau}}\right]$$

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