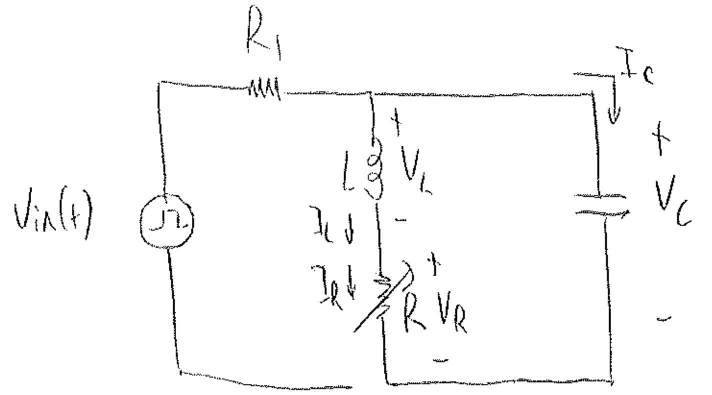
LAB-2 (ECE 232) (continue)



$R_1 = 10k\Omega$

$R = 10k\Omega$ pot

$L = 0.1$ Henry

$C = 0.1\,\mu F$

Differential equations.

$$\frac{d^2V_c}{dt^2} + \left(\frac{1}{R_1C} + \frac{R}{L}\right)\frac{dV_c}{dt} + \left(\frac{R}{R_1LC} + \frac{1}{LC}\right)V_c = \frac{R}{R_1LC}V_{in}(t)$$

characteristic equation $\quad s^2 + \left(\frac{1}{R_1C} + \frac{R}{L}\right)s + \left(\frac{R}{R_1LC} + \frac{1}{LC}\right) = 0$

$$2\alpha = \left(\frac{1}{R_1C} + \frac{R}{L}\right) \qquad w_0^2 = \left(\frac{R}{R_1LC} + \frac{1}{LC}\right)$$

$$2\alpha = 1000 + 10R \qquad w_0^2 = 10^4R + 10^8$$

$$\boxed{\alpha = 500 + 5R} \qquad \boxed{w_0 = 10^2\sqrt{R + 10^4}}$$

When DC input is applied [assume $V_{in}(t) = 1$ Volt]

$V_c(t) = V_{cp} + V_{cn}$ put V_{cp} in diff equation [$V_{cp} = K$ constant]

$$\frac{d^2}{dt^2}K + \left(\frac{1}{R_1C} + \frac{R}{L}\right)\frac{dK}{dt} + \left(\frac{R}{R_1LC} + \frac{1}{LC}\right)K = \frac{R}{R_1LC}\cdot 1 \rightarrow K = \frac{\frac{R}{R_1LC}}{\frac{R}{R_1LC} + \frac{1}{LC}}$$

$$K = \frac{R\,10^4}{R\,10^4 + 10^8} = V_{cp}$$

(for underdamped overdamped and critically-damped cases)

$$\frac{d^2 I_L}{dt^2} + \left(\frac{R}{L} + \frac{1}{R_1 C}\right)\frac{dI_L}{dt} + \left(\frac{1}{LC} + \frac{R}{LR_1C}\right) = \frac{V_{in}}{LR_1C}$$

assume DC input is applied ($V_{in} = 1$ Volt) $I_{Lp} = M$

put I_{Lp} in diff equation

$$\frac{d^2 M}{dt^2} + \left(\frac{R}{L} + \frac{1}{R_1 C}\right)\frac{dM}{dt} + \left(\frac{1}{LC} + \frac{R}{LR_1C}\right)M = \frac{1}{LR_1C}$$

$$M = \frac{\frac{1}{LR_1C}}{\frac{1}{LC} + \frac{R}{LR_1C}} = \frac{10^4}{R10^4 + 10^8}$$

(For underdamped overdamped and critically damped cases)

$V_c(t) = V_{cp} + V_{cn}$ $\qquad$ $I_L(t) = I_{Lp} + I_{Ln}$

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roots of characteristic equation $s_{1,2} = -\alpha \mp \sqrt{\alpha^2 - \omega_0^2}$

$$s_{1,2} = -(500 + 5R) \mp \sqrt{(500+5R)^2 - (10^4 R + 10^8)}$$

For critically-damped case $\alpha = \omega_0$ $\quad (500 + 5R_0) = 10^2\sqrt{R_0 + 10^4}$

$$R_0^2 - 400R_0 - 3990000 = 0 \longrightarrow \text{roots } R_{0_{1,2}} \cong \frac{+400 \mp 4014.97}{2}$$

$\boxed{R_{0_1} \cong 2207}$ $\qquad$ $R_{0_2} = -1807.48$ (not possible)

Hence when $R = R_0 \cong 2207\,\Omega$ (system is critically damped)

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* when

$0 < R < R_0 \qquad 0 < R < 2207$ (the system is underdamped)

$$S_{1,2} = -(500+5R) \mp j\sqrt{(10^6R+10^8)-(500+5R)^2}$$

assume $R = \frac{R_0}{4} = 551.75 \longrightarrow V_{cp} = 0.0522$ Volt

$I_{cp} = 9.4778 \times 10^{-5}$ Amper

$$S_{1,2} = -3255 \mp j\,9742.4$$

$$V_c(t) = V_{cp} + V_{ch} = 0.0522 + e^{-3255t}\left[A_1 \cos(9742.4t) + A_2 \sin(9742.4t)\right]$$

$$I_L(t) = I_{cp} + I_{ch} = 9.4778 \times 10^{-5} + e^{-3255t}\left[B_1 \cos(9742.4t) + B_2 \sin(9742.4t)\right]$$

$\lim_{t\to\infty} V_c(t) = 0.0522$ Volt $\qquad V_R(t) = R\,I_L(t)$

$\lim_{t\to\infty} I_L(t) = 9.4778 \times 10^{-5}$ Amper $\qquad V_L(t) = L\frac{dI_L}{dt}$

$\lim_{t\to\infty} V_R(t) = \lim_{t\to\infty} V_c(t) = 0.0522$ Volt

$\lim_{t\to\infty} V_L(t) = 0$ Volt

** when

$R = R_0 \qquad R = R_0 = 2207\,\Omega \qquad S_{1,2} \cong -11535 \rightarrow V_{cp} = 0.1808$ Volt

$\rightarrow I_{cp} = 8.1920 \times 10^{-5}$ Amper

$$V_c(t) = V_{cp} + V_{ch} = 0.1808 + e^{-11535t}(A_1 t + A_2) \quad \text{(Volt)}$$

$$I_L(t) = I_{cp} + I_{ch} = 8.1920 \times 10^{-5} + e^{-11535t}(B_1 t + B_2) \quad \text{(Amper)}$$

$V_R = R\,I_L(t)$

$V_L = L\frac{dI_L}{dt}(t)$

$\lim_{t\to\infty} V_c(t) = 0.1808 \qquad \lim_{t\to\infty} I_L(t) = 8.1920 \times 10^{-5}$ Amper

$\lim_{t\to\infty} V_R(t) = \lim_{t\to\infty} V_c(t) = 0.1808$ Volt $\qquad \lim_{t\to\infty} V_L(t) = 0$ Volt

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$R = 3R_0 = 6621\,\Omega$ $s_1 = -2571.4$ $s_2 = -64639$ $V_{cp} = 0.3984$ Volt $I_{Lp} = 6.0165\times10^{-5}$ Amper

$$V_c(t) = 0.3984 + A_1 e^{-2571.4t} + A_2 e^{-64639t}$$

$$I_L(t) = 6.0165\times10^{-5} + B_1 e^{-2571.4t} + B_2 e^{-64639t}$$

$$\lim_{t\to\infty} V_c(t) = 0.3984 \text{ Volt} \qquad \lim_{t\to\infty} I_L(t) = 6.0165\times10^{-5} \text{ Amper}$$

$$\lim_{t\to\infty} V_R(t) = \lim_{t\to\infty} R I_L(t) = 0.3984 \text{ Volt.}$$

$$\lim_{t\to\infty} V_L(t) = 0 \text{ Volt}$$

Assume input is $V_{in}(t) \rightarrow$ square wave with $f = 150$ Hz

if $f = 150$ Hz $T = \frac{1}{150} = 6.7$ ms

Compare T with τ values for each case

* Overdamped $s_1 = -2571.4$ $s_2 = -64639$ $\tau_1 = \frac{1}{|s_1|}$ $\tau_2 = \frac{1}{|s_2|}$

$R = 3R_0$

$\tau_1 = 0.3889$ ms $\tau_2 = 0.015471$ ms

$T >> \tau_{1,2}$ [square wave can be thought as a DC input since $T >> \tau_{1,2}$]

Hence $V_c(t)$ and $I_L(t)$ [in general the circuit] will go into steadystate condition

* Critically damped $s_{1,2} = -11535$

$R = R_0$ $\tau_1 = 0.08192\ ms$ $\tau_2 = 0.08192\ ms$

$T = 6.7ms >> \tau_{1,2}$ [square wave can be thought as a DC input since $T >> \tau_{1,2}$]

Hence $V_c(t)$ and $I_c(t)$ will quickly reach to steady-state condition.

* Underdamped case $s_{1,2} = -3255 \mp j\,9742.4$

$R = \frac{R_0}{4} = 551.75$ $\tau_{1,2} = 0.30727ms$ $T = 0.67ms >> \tau_{1,2}$

$$\tau_{1,2} = \frac{1}{|Real(s_{1,2})|}$$

square wave can be thought as a DC input

$V_c(t)$ and $I_c(t)$ will reach steady-state very quickly.