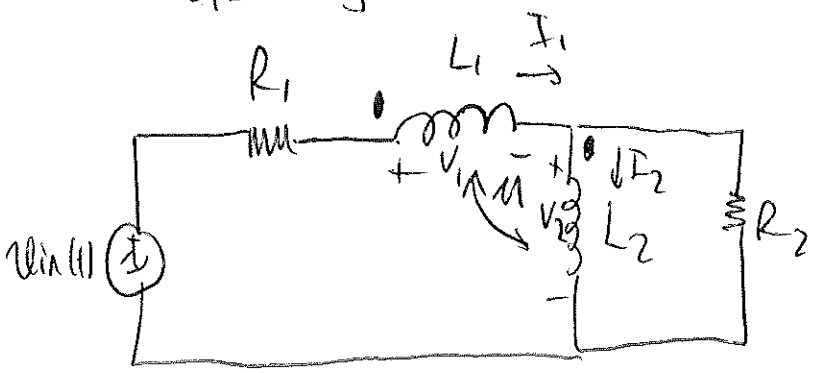
MELE 233 Final 2017-2018 spring

Q1 $v_{in}(t) = 10\cos(t)$ Volt $\rightarrow \bar{V}_{in} = 10$ $(\omega = 1$ rad/sec$)$

$L_1 = 3$ Henry $L_2 = 3$ Henry $M = 1$ Henry $R_1 = 1\Omega$ $R_2 = 3\Omega$



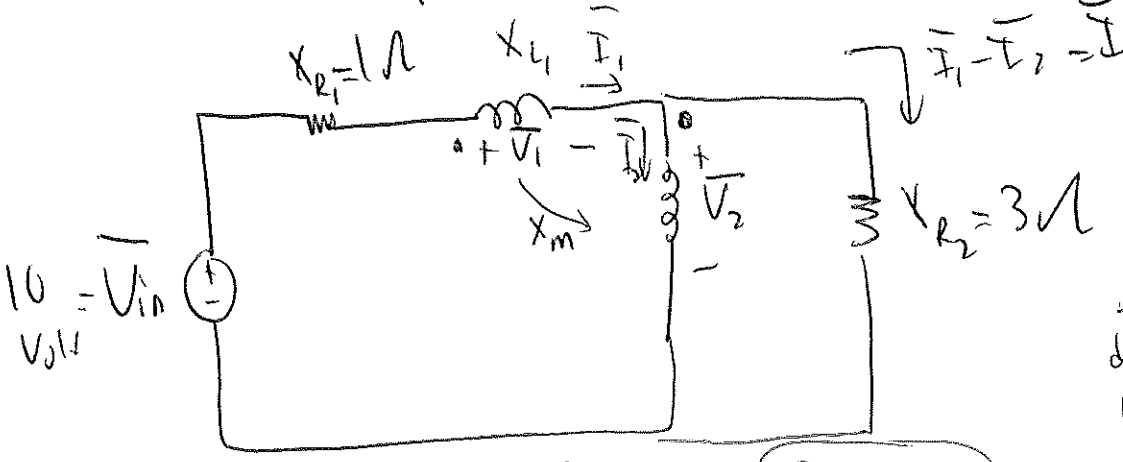
$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} L_1 & M \\ M & L_2 \end{bmatrix} \begin{bmatrix} \frac{dI_1}{dt} \\ \frac{dI_2}{dt} \end{bmatrix}$$

In phasor domain

$\bar{V}_{in} = 10$ Volt $\quad X_{R_1} = R_1 \quad X_{R_2} = R_2$

$$\begin{bmatrix} \bar{V}_1 \\ \bar{V}_2 \end{bmatrix} = \begin{bmatrix} jwL_1 & jwM \\ jwM & jwL_2 \end{bmatrix} \begin{bmatrix} \bar{I}_1 \\ \bar{I}_2 \end{bmatrix}$$

circuit in phasor domain



$\bar{I}_1 - \bar{I}_2 = \bar{I}_3$

10 puan

5 puan

$$\bar{V}_{in} = \bar{I}_1 X_{R_1} + \bar{V}_1 + \bar{V}_2$$

$$10 = \bar{I}_1 \times 1 + \left[jwL_1\bar{I}_1 + jwM\bar{I}_2\right] + \left[jwM\bar{I}_1 + jwL_2\right]\bar{I}_2$$

$$10 = \bar{I}_1 + \left[3j\bar{I}_1 + j\bar{I}_2\right] + \left[j\bar{I}_1 + 3j\bar{I}_2\right)$$

$$10 = \left[1+4j\right]\bar{I}_1 + 4j\bar{I}_2$$

3 puan

$\bar{V}_2 = X_{P2}(\bar{I}_1 - \bar{I}_2)$ (5 puan)

$jw M \bar{I}_1 + jw L_2 \bar{I}_2 = 3(\bar{I}_1 - \bar{I}_2)$ (3 puan)

$(j-3)\bar{I}_1 + (3j+3)\bar{I}_2 = 0$

$\bar{I}_1 = \frac{-(3j+3)\bar{I}_2}{j-3} = \frac{(3j+3)\bar{I}_2}{3-j}$ **

Put ** in *

$10 = [1+4j]\frac{(3+3j)\bar{I}_2}{(3-j)} + 4j\bar{I}_2$

$10 = \frac{[3-12+15j]}{3-j}\bar{I}_2 + 4j\bar{I}_2$

$10(3-j) = [-9+15j]\bar{I}_2 + 4j(3-j)\bar{I}_2$ (3 puan)

$30 - 10j = [-9+15j+12j+4]\bar{I}_2$ $\quad \frac{30-10j}{27j-4} = \bar{I}_2$

$\bar{I}_1 = \frac{3(j+1)}{3-j}\bar{I}_2 = \frac{3(j+1)}{3-j}\frac{30-10j}{27j-5} = \frac{3(j+1)\times 10(3-j)}{3-j(27j-5)}$ (3puan)

$\bar{I}_1 = \frac{30(j+1)}{27j-5}$ $\quad \bar{I}_x = \bar{I}_1 - \bar{I}_2 = \frac{30j+30}{27j-5} - \left[\frac{30-10j}{27j-5}\right]$

$\bar{I}_x = \frac{40j}{27j-5} = \frac{40j^2}{27j^2-5j} = \frac{-40}{-27-5j} = \frac{40}{27+5j} = 40\frac{27-5j}{27^2+5^2}$ A

(3puan)

$P_{R_2} = \frac{1}{2}R_2\bar{I}_x\bar{I}_x^* = \frac{1}{2}\times 3\times\left[40\frac{27-5j}{27^2+5^2}\right]\left[40\frac{27+5j}{27^2+5^2}\right]$ (5puan)

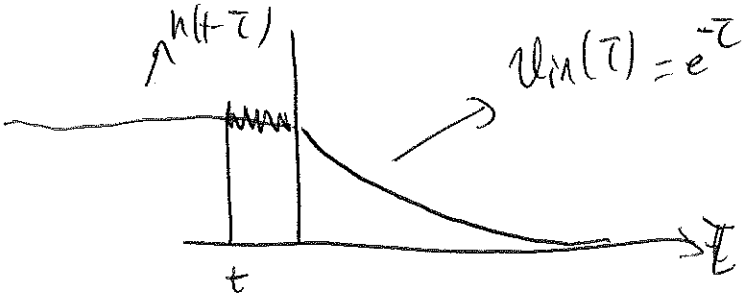
$= \frac{1}{2}\times 3\times\frac{40^2}{27^2+5^2} = \frac{1}{2}\times 3\times\frac{40^2}{27^2+5^2} = \frac{3\times 40^2}{3^2} = 1600$ Watt

(Q2) $h(t) = u(t)$ $\quad v_{in}(t) = e^{-t}$

$$v_{out}(t) = \int_{-\infty}^{t} h(t-\tau)\, v_{in}(\tau)\, d\tau$$

$t < 0$

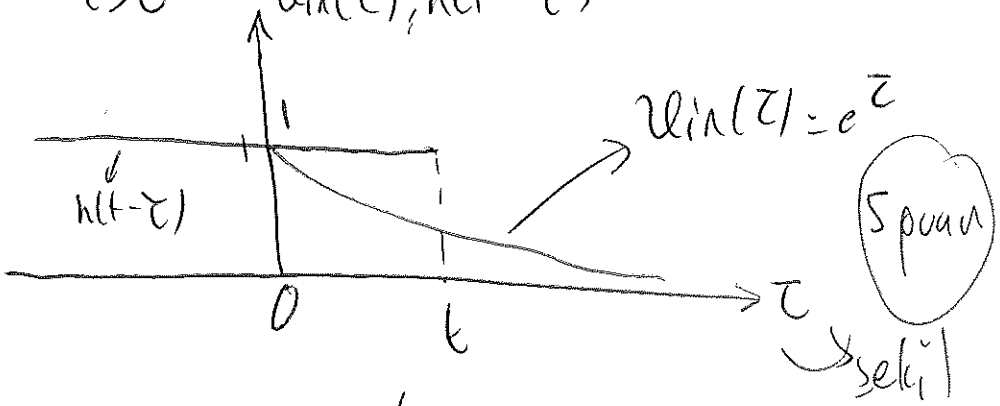
$v_{in}(\tau)$, $h(t-\tau)$



5 puan

$$v_{out}(t) = \int_{-\infty}^{t} h(t-\tau)\, v_{in}(\tau)\, d\tau = 0$$

$t > 0$



Result

$$v_{out}(t) = \begin{cases} 0, & t<0 \\ 1-e^{-t}, & t \geq 0 \end{cases}$$

$$v_{out}(t) = \int_{-\infty}^{t} h(t-\tau)\, v_{in}(\tau)\, d\tau$$

$$= \int_{-\infty}^{0} h(t-\tau)\, v_{in}(\tau)\, d\tau + \int_{0}^{t} h(t-\tau)\, v_{in}(\tau)\, d\tau$$

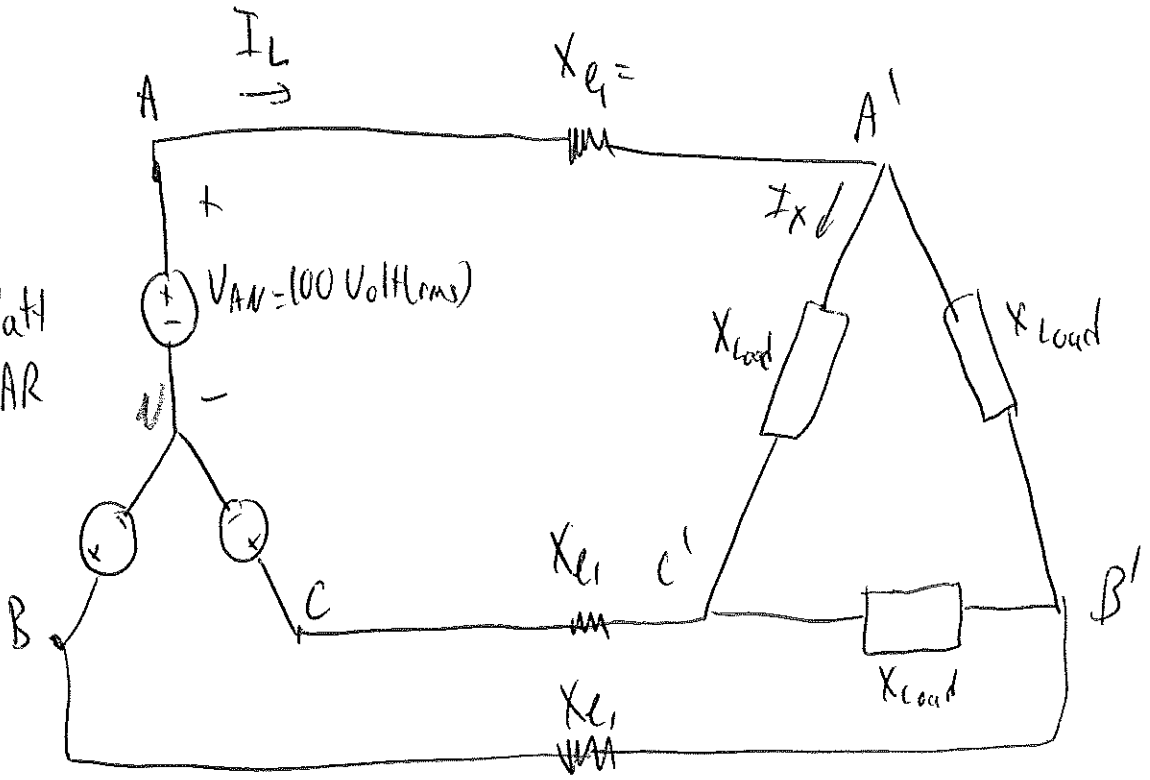
(5 puan)

$$= \int_{-\infty}^{0} 1 \cdot 0 \, d\tau + \int_{0}^{t} 1 \cdot e^{-\tau}\, d\tau$$

$$v_{out}(t) = -e^{-\tau}\Big|_0^t = -\left[e^{-t} - e^{0}\right] = 1 - e^{-t}$$

10 puan

(Q3)



$X_{\ell_1} = 12\Omega$

$P_{source} = 400$ Watt

$Q_{source} = -300$ VAR (C)

$S_{source} = \sqrt{(P_{source})^2 + (Q_{source})^2}$

$|S_{source}| = \sqrt{(400)^2 + (-300)^2} = 500$ VA (2 pvan)

$3|V_{AN}||I_L| = |S_{source}|$

$|I_L| = \frac{500}{3\times 100} = \frac{5}{3}$ Amper (2 pvan)

$|V_{AC}| = 100\sqrt{3}$ Volt (rms) (2 pvan)

$P_{line} = 3|I_L|^2 Re\{X_{\ell_1}\}$ (2 pvan)

$= 3\left(\frac{5}{3}\right)^2 \cdot 12 = 100$ Watt

$Q_{line} = 3|I_L|^2 Im\{X_{\ell_1}\}$ (2 pvan)

$= 0$ VAR

$P_{Load} = P_{source} - P_{line} = 400 - 300 = 300$ Watt (2 pvan)

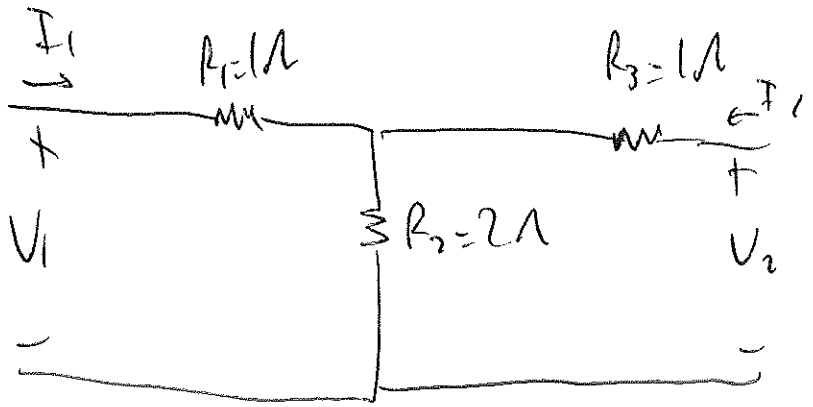
$Q_{Load} = Q_{source} - Q_{line} = -300 - 0 = -300$ VAR (2 pvan)

$|S_{Load}| = \sqrt{P_{load}^2 + Q_{load}^2} = 300\sqrt{2}$ VA // (2 pvan)

$|V_{A'C'}| = \frac{|S_{load}|}{\sqrt{3}\,|I_L|} = \frac{300\sqrt{2}}{\sqrt{3}\,\frac{5}{3}} = 60\sqrt{6}$ Volt (2 pvan)

$|I_x| = \frac{|I_L|}{\sqrt{3}} = \frac{5}{3\sqrt{3}}$ Ampere (rms) (2 pvan)

Q4:



$V_1 = R_1 I_1 + R_2 (I_1 + I_2)$ $\quad V_2 = R_3 I_2 + R_2 (I_1 + I_2)$

$V_1 = (R_1 + R_2) I_1 + R_2 I_2$ $\quad V_2 = R_2 I_1 + (R_2 + R_3) I_2$

$V_1 = 3 I_1 + 2 I_2$ $\quad V_2 = 2 I_1 + 3 I_2$

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 3 & 2 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$